

Answer script

Algebraic ratio and proportion

(Barkatul alam tipu)

Answer to the qo,no1/a

$$\frac{2x-y}{x-2y} = \frac{a}{b}$$

Or, $a(x-2y) = b(2x-y)$ { cross multiply}

Or, $ax-2ay=2bx-by$

Or, $ax-2bx=2ay-by$

Or, $x(a-2b)=y(2a-b)$

$$\frac{x}{y} = \frac{(2a-b)}{(a-2b)} \text{ Ans.}$$

Answer to the qo. No 1/b

Given $x = \frac{3}{5}$,

Or, $\frac{1}{x} = \frac{5}{3}$ [Invertendo]

Or, $\frac{1+x}{1-x} = \frac{5+3}{5-3}$ [cmponando-divinando]

Or, $\frac{1+x}{1-x} = \frac{8}{2}$

Or, $\frac{1+x}{1-x} = 4$

Or, $\frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{4}$ [square root]

Or, $\frac{\sqrt{1+x}}{\sqrt{1-x}} = \pm 2$

$$\text{Or, } \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} = \frac{2+1}{2-1} \text{ again, } \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} = \frac{-2+1}{-2-1} \text{ [cmponando-divinando]}$$

$$\text{Or, } \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} = 3 \text{ again, } \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} = \frac{1}{3} \text{ [ans]}$$

Answer 1/c

$$\text{Given, } \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} = p$$

$$\text{Or, } \frac{\sqrt{1+x}+\sqrt{1-x}+\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}-\sqrt{1+x}+\sqrt{1-x}} = \frac{p+1}{p-1} \text{ [cmponando-divinando]}$$

$$\text{Or, } \frac{2\sqrt{1+x}}{2\sqrt{1-x}} = \frac{p+1}{p-1}$$

$$\text{Or, } \frac{1+x}{1-x} = \frac{(p+1)^2}{(p-1)^2} \text{ [squaring both sides]}$$

$$\text{Or, } \frac{1+x+1-x}{1+x-1+x} = \frac{(p+1)^2+(p-1)^2}{(p+1)^2-(p-1)^2} \text{ [cmponando-divinando]}$$

$$\text{Or, } \frac{2}{2x} = \frac{p^2+2p+1+p^2-2p+1}{p^2+2p+1-p^2+2p-1}$$

$$\text{Or, } \frac{2}{2x} = \frac{2p^2+2}{4p}$$

$$\text{Or, } \frac{1}{x} = \frac{2(p^2+1)}{4p}$$

$$\text{Or, } x(p^2 + 1) = 2p \text{ [cross-multiply]}$$

$$\text{Or, } p^2 + 1 = \frac{2p}{x}$$

$$\text{Or, } p^2 - \frac{2p}{x} + 1 = 0 \text{ [proved]}$$

Answer to the question no; 2/a

$$\text{Given, } a = \frac{\sqrt[3]{m+1}+\sqrt[3]{m-1}}{\sqrt[3]{m+1}-\sqrt[3]{m-1}}$$

$$\text{Or, } \frac{a+1}{a-1} = \frac{\sqrt[3]{m+1}+\sqrt[3]{m-1}+\sqrt[3]{m+1}-\sqrt[3]{m-1}}{\sqrt[3]{m+1}+\sqrt[3]{m-1}-\sqrt[3]{m+1}+\sqrt[3]{m-1}} \text{ [cmponando-divinando]}$$

$$\text{Or, } \frac{a+1}{a-1} = \frac{2\sqrt[3]{m+1}}{2\sqrt[3]{m-1}}$$

$$\text{Or, } \frac{a+1}{a-1} = \frac{\sqrt[3]{m+1}}{\sqrt[3]{m-1}} \text{ [answer]}$$

Answer to the question no; 2/b

Prove that $a^3 - 3ma^2 + 3a - m = 0$

$$\frac{a+1}{a-1} = \frac{\sqrt[3]{m+1}}{\sqrt[3]{m-1}} \text{ [From-a]}$$

$$\text{Or, } \frac{(a+1)^3}{(a-1)^3} = \frac{(\sqrt[3]{m+1})^3}{(\sqrt[3]{m-1})^3} \text{ , [cubing both sides]}$$

$$\text{Or, } \frac{a^3+3a^2+3a+1}{a^3-3a^2+3a-1} = \frac{m+1}{m-1}$$

$$\text{Or, } \frac{a^3+3a^2+3a+1+a^3-3a^2+3a-1}{a^3+3a^2+3a+1-a^3+3a^2-3a+1} = \frac{m+1+m-1}{m+1-m+1} \text{ [cmponando-divinando]}$$

$$\text{Or, } \frac{2a^3+6a}{6a^2+2} = \frac{2m}{2}$$

$$\text{Or, } \frac{2[a^3+3a]}{2(3a^2+1)} = m$$

$$\text{Or, } a^3 + 3a = 3a^2m + m$$

$$\text{Or, } a^3 - 3ma^2 + 3a - m = 0 \text{ [proved]}$$

Answer to the question no; 2/c

If $\frac{bz-cy}{a} = \frac{cx-az}{b} = \frac{ay-bx}{c}$ then prove that, $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$

Let, $\frac{bz-cy}{a} = \frac{cx-az}{b} = \frac{ay-bx}{c} = k$ [constant]

$$bz-cy = ak \dots \dots \dots (i) \times a$$

$$cx - az = bk \dots \dots \dots (ii) \times b \text{ [multiply equation, i, ii, iii by a, b, c]}$$

$$ay - bx = ck \dots \dots \dots (iii) \times c$$

$$abz - acy = a^2 k \dots \dots \dots (iv)$$

$$bcx - abz = b^2 k \dots \dots \dots (v)$$

$$\underline{acy - bcx = c^2 k \dots \dots \dots (vi)}$$

$$0 = k(a^2 + b^2 + c^2) \text{ [adding]}$$

$$\text{Or, } k = 0$$

$$\text{So, } bz - cy = 0$$

$$\text{Or, } bz = cy$$

$$\text{Or, } \frac{bz}{bc} = \frac{cy}{bc} \text{ [dividing by ,bc]}$$

$$\text{Or, } \frac{z}{c} = \frac{y}{b} \text{ similarly, } \frac{x}{a} = \frac{y}{b} \text{ [proved]}$$

Answer to the question no; 3/a

$$\text{Given, } \frac{1}{p} + \frac{1}{q} = \frac{8}{x}$$

$$\text{Or, } \frac{p+q}{pq} = \frac{8}{x}$$

$$\text{Or, } x(p + q) = 8pq \text{ [cross-multiply]}$$

$$\text{Or, } x = \frac{8pq}{p+q} \text{ [answer]}$$

Answer to the question no; 3/b

$$\text{Find the value of } \frac{x+4p}{x-4p} + \frac{x+4q}{x-4q}$$

$$x = \frac{8pq}{p+q} \text{ [from-a]}$$

$$\text{or, } \frac{x}{4p} = \frac{2q}{p+q} \text{ [dividing by ,4p]}$$

$$\text{or, } \frac{x+4p}{x-4p} = \frac{2q+p+q}{q-p} \text{ [cmponando-divinando]}$$

$$\text{or, } \frac{x+4p}{x-4p} = \frac{3q+p}{q-p} \text{(i)}$$

$$\text{Again, } x = \frac{8pq}{p+q}$$

$$\text{or, } \frac{x}{4q} = \frac{2p}{p+q} \text{ [dividing by ,4q]}$$

$$\text{or, } \frac{x+4q}{x-4q} = \frac{2p+p+q}{p-q} \text{ [cmponando-divinando]}$$

$$\text{or, } \frac{x+4q}{x-4q} = \frac{3p+q}{p-q} \text{(ii)}$$

Now adding equation (i) and (ii)

$$\frac{x+4p}{x-4p} + \frac{x+4q}{x-4q} = \frac{3q+p}{q-p} + \frac{3p+q}{p-q}$$

$$\text{Or, } \frac{x+4p}{x-4p} + \frac{x+4q}{x-4q} = \frac{3q+p}{q-p} - \frac{3p+q}{q-p}$$

$$\text{Or, } \frac{x+4p}{x-4p} + \frac{x+4q}{x-4q} = \frac{3q+p-3p-q}{q-p}$$

$$\text{Or, } \frac{x+4p}{x-4p} + \frac{x+4q}{x-4q} = \frac{2q-2p}{q-p}$$

$$\text{Or, } \frac{x+4p}{x-4p} + \frac{x+4q}{x-4q} = \frac{2(q-p)}{(q-p)}$$

$$\text{Or, } \frac{x+4p}{x-4p} + \frac{x+4q}{x-4q} = 2 \text{ [answer]}$$

Answer to the question no; 3/c

From the stem, $\frac{p^2+q^2}{q^2+r^2} = \frac{(p+q)^2}{(q+r)^2}$

Or, $\frac{p^2+q^2}{(p+q)^2} = \frac{q^2+r^2}{(q+r)^2}$ [alternando]

Or, $\frac{|p+q|^2}{p^2+q^2} = \frac{|q+r|^2}{q^2+r^2}$ [Invertando]

or $\frac{(p+q)^2 - p^2 - q^2}{p^2+q^2} = \frac{(q+r)^2 - q^2 - r^2}{q^2+r^2}$ [divinando]

or, $\frac{2pq}{p^2+q^2} = \frac{2qr}{q^2+r^2}$

or, $\frac{p}{p^2+q^2} = \frac{r}{q^2+r^2}$ [dividing by 2q]

or, $p[q^2 + r^2] = r[p^2 + q^2]$ [cross-multiply]

or, $pq^2 + pr^2 = rp^2 + rq^2$

or, $pq^2 - rq^2 = rp^2 - pr^2$

or, $q^2(p - r) = pr(p - r)$

or, $q^2 = pr$

or, $\frac{q^2}{pq} = \frac{pr}{pq}$

or, $\frac{q}{p} = \frac{r}{q}$

therefore p, q, r are continued proportion [proved]

